equally simple, $\dfrac{M_{\overset{1}{x+n.y+n}}+M_{\overline{x+n.y+n}}^{\;\;1}}{N_{x.y}}$. If the number of premiums be limited to n, of course $N_{x.y}-N_{x+n.y+n}$ must be substituted in the denominator.

But although these tables considerably lessen the labour of solving such problems as the above, their full value cannot be appreciated, unless we compare the simplicity of the operations now required in the finding of values in which the element of survivance is introduced (which by them is reduced to a simple division), with the laborious methods which were formerly rendered necessary.

Hoping that these few remarks may turn the attention of such of your readers as have not already examined Mr. Chisholm's volumes, to the great facilities which they offer to those who are engaged in intricate calculations.

I am, Sir,
Your obedient servant,
W. F. B.

Edinburgh, 27th Nov., 1858.

DEMONSTRATION OF FORMULÆ FOR VALUE OF AN ENDOWMENT ASSURANCE.

To the Editor of the Assurance Magazine.

SIR,—It is very well known that an assurance payable at a given age or at previous death—commonly called an "endowment assurance"—bears a close analogy to an ordinary whole term assurance. This analogy has been pointed out by Mr. Gray (*Tables of Life Contingencies*, art. 233); but he has not given any of the formulæ for the assurance in question. The subject has been also touched upon by yourself—*Assurance Magazine*, vol. i., p. 332—where the proper formulæ are given and demonstrated; a formula is also supplied by Mr. Hardy in his *New and General Notation*, p. 43. The following convenient practical rule is easily seen to follow at once from the reasoning in the passage in the *Assurance Magazine* just referred to:—"To find the annual or single premium for an endowment assurance payable at the age $m+t$ on a life now aged m, calculate the temporary annuity for $t-1$ years on the life m, and enter Orchard's Tables with the result, in just the same way as for an ordinary whole term assurance."

As assurances of the kind in question are not at all uncommon, being granted by most Insurance Companies, and the subject is therefore of some practical importance, I have thought that the following independent proof of the above rule will be interesting to the readers of the *Assurance Magazine*.

Since an endowment assurance on a life m, payable at age $m+t$, is equivalent to a term assurance for t years and an endowment at the end of t years, the single premium for it will be $\dfrac{M_m-M_{m+t}+D_{m+t}}{D_m}$. The annual premium is got by substituting $N_{m-1}-N_{m+t-1}$ for D_m in the denominator, and is therefore $\dfrac{M_m-M_{m+t}+D_{m+t}}{N_{m-1}-N_{m+t-1}}$.

But, $$M_m=D_m-(1-r)N_{m-1},$$
$$M_{m+t}=D_{m+t}-(1-r)N_{m+t-1};$$
therefore, $$M_m-M_{m+t}+D_{m+t}=D_m-(1-r)(N_{m-1}-N_{m+t-1});$$

and the annual premium is consequently equal to

$$\frac{D_m}{N_{m-1}-N_{m+t-1}}-(1-r), \quad \text{or } \frac{1}{1+a_{m\overline{t-1}|}}-(1-r),$$

—which proves the rule stated above for the annual premium. It is also at once seen that the single premium is

$$1-(1-r)\frac{N_{m-1}-N_{m+t-1}}{D_m}=1-(1-r)\left(1+a_{m\overline{t-1}|}\right);$$

—from which a similar conclusion follows.

It may here be noticed, that the formulæ and the process of solution and proof are exactly analogous when two lives are involved, as in some cases which have recently occurred to myself. Thus, an endowment assurance on two lives, m, n, will be an assurance of a sum payable at the end of a certain number (t) of years, or upon the death of either of the lives, m, n, should that occur within the t years; and is therefore the sum of a joint life term assurance and a joint life endowment. The formula for the single premium will consequently be $\frac{M_{m.n}-M_{m+t.n+t}+D_{m+t.n+t}}{D_{m.n}}$, which, as above, is equal to

$$1-(1-r)\frac{N_{m-1.n-1}-N_{m+t-1.n+t-1}}{D_{m.n}}, \quad \text{or } 1-(1-r)\left(1+a_{m.n\overline{t-1}|}\right).$$

In order, then, to find the premium for such an assurance, it will suffice, as in the case of a single life, to enter Orchard's Tables with the annuity on the joint lives for a term of $t-1$ years.

In the last Number of the *Assurance Magazine*, I gave a proof of a formula for a term insurance on two joint lives, and stated that it was not to be found in the treatises on Life Insurance. It has since been pointed out to me by Mr. Laundy, of the Eagle Insurance Company, that the formula in question is given, but without any demonstration, in *Jones on Annuities*, vol. ii., p. v.; also in a paper by Professor De Morgan in the *Companion to the Almanac* for the year 1842, p. 2.

That I may not be indebted to Mr. Laundy without making him some return, I will supply what seems to me an omission in his communication which precedes my own, in the last Number of the *Assurance Magazine*. In treating of the question—"For what amount should a policy free from future payment of premium be given, in consideration of the surrender of an existing assurance?" he does not state how he would deduce a practical value from the theoretical one given by his formula. I presume he would take off some percentage; but the following seems the more theoretically correct method of proceeding. Let the full value of the policy be taken, and the corresponding reversion be calculated by an "Office single premium." Thus, if A'_m be the Office single premium at age m, we shall have $A'_m=(1+a_m)P'_m$, where P'_m is the annual premium charged by the Office at the age m. Then the full value of the policy being $\Sigma=(1+a_{m+n})(P_{m+n}-P_m)$, where P_m, P_{m+n}, are the net premiums at the ages m, n, the amount of the free policy will be $\frac{\Sigma}{A'_{m+n}}=\frac{P_{m+n}-P_m}{P'_{m+n}}$. When the premiums charged by the Office are obtained from the net premiums by adding a percentage, the amount of the free policy, as given

by this formula, will be in the same ratio to the theoretical amount that the net premium is to the premium charged; but this will not be so when, as is often the case, the premiums are formed in some other way.

I remain, Sir,
Your obedient servant,
T. B. SPRAGUE.

Liverpool and London Insurance Company,
20, *Poultry, London,*
4*th December*, 1858.

FORMULÆ FOR THE PREMIUM FOR A TERM ASSURANCE ON TWO JOINT LIVES.

To the Editor of the Assurance Magazine.

SIR,—In the last Number of your *Journal*, your talented correspondent, Mr. T. B. Sprague, remarks, that the formulæ given by him for the single and annual premium of a temporary assurance on the joint lives of two persons may probably be *new* to the readers of the *Assurance Magazine*, as they are not given in David Jones' *Treatise on Annuities*, or in any other work with which he is acquainted. It is true that the *commutation* formulæ for joint lives do not appear in the first volume of Jones, but your readers will find them capitulated on pages 5 and 6 of the second volume of that work, also in a paper on "Life Contingencies," by Professor De Morgan, in the *Companion to the British Almanack*, for the year 1842, where the subject is treated in a very elegant and masterly style.

I know from experience that the formulæ in question do not frequently occur in practice; but it is well to know how to investigate them, inasmuch that the exact expressions are always preferable to the approximate methods, when, as in the present case, the results are not too complicated for practical use.

The mode of investigation adopted by Mr. Sprague, in your last Number, is very neat and concise, but as that method is based upon Jones' formulæ,

$$(\mathrm{A})_{\overline{t}|}=r\left\{1+(a)_{\overline{t-1}|}\right\}-(a)_{\overline{t}|},$$

the demonstration of which may be considered the most difficult part of the question, it has occurred to me that the following solution, investigated from first principles, may be acceptable to your readers.

I regret that my time has been so much occupied during the last quarter as to prevent me from continuing my paper on "Finite Differences" in the present Number; but I will, if possible, resume the subject in the *Journal* for April next.

I am, Sir,
Your obedient servant,
WM. CURTIS OTTER, F.R.A.S.

PROBLEM.

Required the single and annual premium for the assurance of £1 payable on the death of the first of two lives, A ***and*** B, ***aged*** x ***and*** y, ***provided such death takes place before the expiration of*** t ***years.***